Modified Feynman ratchet with velocity-dependent fluctuations

Jack Denur
Electric & Gas Technology, Inc.
13636 Neutron Road
Dallas, Texas 75244-4410

Abstract. The randomness of Brownian motion at thermodynamic equilibrium can be spontaneously broken by velocity-dependence of fluctuations, i.e., by dependence of values or probability distributions of fluctuating properties on Brownian-motional velocity. Such randomness-breaking can spontaneously obtain via interaction between Brownian-motional Doppler effects — which manifest the required velocity-dependence — and system geometrical asymmetry. A *non*random walk is thereby spontaneously superposed on Brownian motion, resulting in a systematic net drift velocity *despite* thermodynamic equilibrium. The time evolution of this systematic net drift velocity — and of velocity probability density, force, and power output — is derived for a modified Feynman-ratchet model. We show that said spontaneous randomness-breaking, and consequent systematic net drift velocity, imply: bias from the Maxwellian of the system's velocity probability density, the force that tends to accelerate it, and its power output. Maximization, especially of power output, is discussed. Uncompensated decreases in total entropy, in violation of the second law of thermodynamics, are thereby implied.

I. VELOCITY-DEPENDENT MODIFIED FEYNMAN RATCHET

The Zhang [1] formulation of the second law of thermodynamics (second law) states that no spontaneous momentum flow is possible at thermodynamic equilibrium (TEQ). By *spontaneous*, it is meant: (a) with*out* cost in negentropy or free energy; and (b) *robust*, i.e., capable of withstanding dissipation, of surviving disturbances, and of generating (regenerating) itself if initially nonexistent (if destroyed). This implies that no systematic motion — most generally, no systematic process — is possible at TEQ. (Systematic processes generated and maintained spontaneously *despite TEQ* violate the second law. By contrast, systematic *non*spontaneous *non*dissipative processes do not violate the second law, but merely imply that TEQ has not been *completely* realized [2].) Feynman's classic ratchet and pawl [3] elucidates the Zhang [1] formulation of the second law. Recently, various formulations of the second law have been challenged, mainly in the quantum regime [4–6], but also classically [7].

In this paper, we show that velocity-dependent fluctuations (but not fluctuations in general) challenge the second law in the classical regime. (Our challenge may also obtain in the quantum regime, but this aspect is not studied herein.) Our challenge is most self-evident with respect to the Zhang [1] formulation of the second law, but a challenge to the Zhang [1] formulation of the second law also is a challenge to all other formulations thereof. Feynman's ratchet [3] is modified to the minimum extent necessary to ensure that velocity-dependence of fluctuations can spontaneously break the randomness of its Brownian motion at TEQ — spontaneously superposing a *non*random walk on its Brownian motion and hence challenging the second law. This minimally-modified Feynman ratchet, a disk-and-pawl system (DP), illustrated in Fig. 1, will now be described.

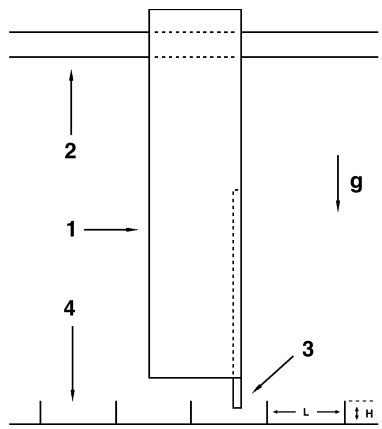

FIGURE 1. Modified Feynman ratchet with velocity-dependent fluctuations

In the right-handed Cartesian coordinate system of Fig. 1, the $+X$, $+Y$, and $+Z$ directions are to the right, into the page, and upwards, respectively. The Brownian motion of the disk 1 of mass m' (shown edge-on in Fig. 1) is constrained to be X-directional by the frictionless guide 2. The pawl 3 of mass m is in a vertical groove within the $+X$ disk face, wherein — in addition to its X-directional Brownian motion in lockstep with the disk as part of the combined disk-and-pawl system DP — it also has Z-directional Brownian motion relative to the disk per se. The DP's total mass is $M = m' + m \gg m$. Each peg 4 is of Z-directional height H and is separated from adjacent pegs by X-directional distance L. The DP, and the entire system, is at TEQ with equilibrium blackbody radiation (EBR) at temperature T. L is small enough so that changes in the DP's X-directional Brownian-motional velocity V occur, essentially, *only at* pawl-peg bounces, and *not* via DP-EBR X-directional momentum exchanges *between* pawl-peg bounces [8]; yet (for simplicity) large compared with the combined pawl-plus-peg X-directional thickness. The pawl's altitude Z is taken as $Z = 0$ when its lower tip rests on the floor of the peg row 4. A uniform gravitational field g is attractive downwards.

Corresponding to V, to first order in V/c, Doppler-shifted EBR at temperature [9]

$$T(V, \alpha) = T(1 + \frac{V \cos\alpha}{c}) \tag{1}$$

impinges on the $+X$ disk face (including the pawl) at angle α from the $+X$ direction — at a rate proportional both to the differential solid angle $2\pi \sin\alpha d\alpha$, and by Lambert's cosine law, to $\cos\alpha$ [9]. The pawl, being in the $+X$ disk face, "sees" EBR impinging *only* from directions with $+X$ components (except for its lower tip — of *negligible* size compared with the *entire* pawl even at maximum protrusion, i.e., even at $Z = 0$ — when said tip protrudes below the disk). Thus, averaging over the range $0 \leq \alpha \leq \pi/2$ [9],

$$T(V) = \langle T(V, \alpha) \rangle = \frac{\int_0^{\pi/2} T(1 + \frac{V \cos \alpha}{c}) \sin \alpha \cos \alpha d\alpha}{\int_0^{\pi/2} \sin \alpha \cos \alpha d\alpha} = T(1 + \frac{2V}{3c}). \qquad (2)$$

The DP's thermal response time is sufficiently short that $T(V)$ is the temperature, corresponding to V having a given value, of the $+X$ disk face (including the pawl) *itself*, and *not* merely of Doppler-shifted EBR "seen" thereby [8]. [Of course, for the $-X$ disk face [8], let $+ \longrightarrow -$ in (1) and (2).]

In accordance with the Boltzmann distribution, applying (2), and defining $A \equiv mgH/kT$: the *conditional* probability [10] $P(Z > H|V)$ that the pawl, of weight mg, can attain sufficient altitude $Z > H$ to jump the pegs — and hence *not* to impede the DP's X-directional Brownian motion — given V, is

$$P(Z > H|V) = \exp[-mgH/kT(V)] = \exp\{-mgH/[kT(1 + \frac{2V}{3c})]\}$$
$$\equiv \exp[-A/(1 + \frac{2V}{3c})] = (1 + \frac{2AV}{3c})e^{-A}. \qquad (3)$$

The last step of (3) is correct to first order in V/c, and is justified because $|V| \ll c$ for all values of V that have nonnegligible probabilities of occurrence.

By (3), $P(Z > H|V)$ is slightly greater when $V > 0$ than when $V < 0$. Hence, *despite TEQ*, the *velocity-dependence* of $P(Z > H|V)$ spontaneously superposes a *non*random walk in the $+X$ (Forward) direction on the DP's Brownian motion — challenging the second law.

Note that $T(V, \alpha)$, $T(V)$, Z, and $P(Z > H|V)$ manifest velocity-dependent fluctuations. By contrast, T, H, L, g, m', m, $M = m' + m \gg m$, and $A \equiv mgH/kT$ are parameters, fixed in any one given (thought) experiment.

II. MARKOVIAN TIME EVOLUTION

By (3), the *conditional* probability [10] F (R) of $Z > H$ obtaining given Forward (Reverse) DP Brownian motion in the $+X$ $(-X)$ direction at $V = +|V|$ $(V = -|V|)$ is that given the $+$ $(-)$ signs in (4):

$$F\ (R) \equiv P(Z > H|V = \pm|V|) \equiv P(>|\pm) = (1 \pm \frac{2A|V|}{3c})e^{-A}. \qquad (4)$$

The states $Z > H$, $Z < H$, $V = +|V|$, and $V = -|V|$ are denoted as $>$, $<$, $+$, and $-$, respectively. (Since Z and V are *continuous* random variables, the *point* values $Z = H$ and $V = 0$ each has *zero* probability measure of occurrence.) Given $V = \pm|V|$, immediately preceding any pawl-peg interaction, the DP is in *one* of the four states $> +$, $> -$, $< +$, or $< -$; the former two states implying that this interaction will be a pawl-over-peg jump and the latter two that it will be a pawl-peg bounce. Immediately following a jump (bounce), $\operatorname{sgn} V$ is unchanged (reversed).

Let enclosure within single (double) angular brackets denote expectation value over any *one* given $\pm|V|$ pair (over *all* V). We study our system's time evolution, given $V = \pm|V|$, in discrete time-steps of $\Delta t = L/|V|$ that separate pawl-peg interactions, with time N immediately preceding the $(N+1)st$ pawl-peg interaction. TEQ, i.e., *maximum initial total entropy*, implies that *initially*, at $N = 0$,

$$\begin{aligned} & P(+)_0 = P(-)_0 = P(V)_0 = \frac{1}{2} \\ \Longleftrightarrow \quad & \langle V \rangle_0 = |V|[P(+)_0 - P(-)_0] = 0 \implies \langle\langle V \rangle\rangle_0 = 0. \end{aligned} \tag{5}$$

The expression in (5) for $\langle V \rangle_0$ is true for *all* $\pm|V|$ pairs, hence implying that for $\langle\langle V \rangle\rangle_0$.

Given $V = \pm|V|$ and $P(+)_N + P(-)_N = P(> |\pm) + P(< |\pm) = 1$, said time evolution is a two-state discrete-time Markov chain [11] with (a) states $+$ and $-$, and (b) the following *conditional* transition probabilities: $P[(+)_N|(+)_{N-1}] = P(> |+) = F$, $P[(-)_N|(-)_{N-1}] = P(> |-) = R$, $P[(-)_N|(+)_{N-1}] = P(< |+) = 1 - F$, and $P[(+)_N|(-)_{N-1}] = P(< |-) = 1 - R$. For all $N \geq 0$, we obtain [11]

$$P(+)_N = [2(1-R) - (F-R)(F+R-1)^N]/[2(2-F-R)] = 1 - P(-)_N. \tag{6}$$

Applying (6) [which includes $P(+)_N + P(-)_N = 1$] and (4) yields

$$\begin{aligned} \langle V \rangle_N &= |V|[P(+)_N - P(-)_N] = |V|[2P(+)_N - 1] \\ &= |V|(F-R)[1-(F+R-1)^N]/(2-F-R) \\ &= (2V^2/3c)A[1-(2e^{-A}-1)^N]/(e^A - 1). \end{aligned} \tag{7}$$

By (7), $\langle V \rangle_N$ is antisymmetric in F and R; hence, taking $F \geq R \implies \langle V \rangle_N \geq 0$ as per (4) and (7) entails *no* loss of generality.

By $P(+)_N + P(-)_N = 1$, the first line of (7), (5), and the sentence immediately following (7), a simpler alternative to (6) is

$$\begin{aligned} & P(\pm)_N = P(V = \pm|V|)_N = \frac{1}{2}(1 \pm \frac{\langle V \rangle_N}{|V|}) = P(V)_0(1 \pm \frac{\langle V \rangle_N}{|V|}) \\ \implies \quad & P(V)_N = P(V)_0(1 + \frac{\langle V \rangle_N}{V}). \end{aligned} \tag{8}$$

Considering any *one* given $\pm|V|$ pair, $P(V)_0 = \frac{1}{2}$. By contrast, considering *all* V, $P(V)_0 = P(V)_{\mathrm{mw}} = (M/2\pi kT)^{1/2}\exp(-MV^2/2kT)$, the one-dimensional Maxwellian velocity probability density.

By Newton's second law, (7), and (4), the force f that tends to accelerate the DP towards the $+X$ direction and DP power output P^* (not to be confused with probability P) at the $(N+1)st$ pawl-peg interaction, i.e., at the $N \longrightarrow N+1$ transition, given $V = \pm|V|$, are, respectively,

$$\begin{aligned} \langle f \rangle_{N.5} &= M(\langle V \rangle_{N+1} - \langle V \rangle_N)/\Delta t = M(\langle V \rangle_{N+1} - \langle V \rangle_N)/(L/|V|) \\ &= \frac{MV^2(F-R)(F+R-1)^N}{L} = \frac{4M|V|^3 A e^{-A}(2e^{-A}-1)^N}{3Lc} \end{aligned} \tag{9}$$

and

$$\begin{aligned}\langle P^* \rangle_{N.5} &= \langle fV \rangle_{N.5} = \langle f \rangle_{N.5}[(\langle V \rangle_N + \langle V \rangle_{N+1})/2] \\ &= \frac{M|V|^3(F-R)^2(F+R-1)^N[2-(F+R)(F+R-1)^N]}{2L(2-F-R)} \\ &= \frac{8M|V|^5A^2e^{-A}(2e^{-A}-1)^N[1-e^{-A}(2e^{-A}-1)^N]}{9Lc^2(e^A-1)}. \end{aligned} \tag{10}$$

The second step of (10) is justified because $(\langle V \rangle_N + \langle V \rangle_{N+1})/2$ is *in*dependent of whether state $> +$, $> -$, $< +$, or $< -$ obtains at the $(N+1)st$ pawl-peg interaction, i.e., at the $N \longrightarrow N+1$ transition. [12,13].

Time evolution to final steady state is monotonic and asymptotic if $0 < F+R-1 < 1 \implies \ln 2 > A > 0$, diminishing-oscillatory if $-1 < F+R-1 < 0 \implies \infty > A > \ln 2$, and complete at $N = 1$ if $F+R-1 = 0 \implies A = \ln 2$.

Maxima are: $\langle V \rangle_{N,\max} = \langle V \rangle_\infty |(A \longrightarrow 0) = 2V^2/3c$, $|P(V)_N - P(V)_0|_{\max} = |P(V|A \longrightarrow 0)_\infty - P(V)_0| = 2P(V)_0|V|/3c$, $\langle f \rangle_{N.5,\max} = \langle f \rangle_{0.5}|(A=1) = 4M|V|^3/3ecL$, and $\langle P^* \rangle_{0.5,\max} = \langle P^* \rangle_{0.5}|(A=1) = 8M|V|^5/[(3ec)^2L]$. Equal and/or higher maxima and corresponding A — if any exist — of $\langle P^* \rangle_{N.5}$ for $N \geq 1$ can be found numerically.

Averaging V^2, $|V|^3$, and $|V|^5$ in the respective last terms of (7), (9), and (10) over $P(V)_{\text{mw}}$ yields $\langle\langle V^2 \rangle\rangle = kT/M$, $\langle\langle |V|^3 \rangle\rangle = [2(kT/M)^3]^{1/2}$, and $\langle\langle |V|^5 \rangle\rangle = 8[2(kT/M)^5]^{1/2}$, respectively; and hence the respective expectation values $\langle\langle V \rangle\rangle_N$, $\langle\langle f \rangle\rangle_{N.5}$, and $\langle\langle P^* \rangle\rangle_{N.5}$ over *all* V — as well as, via the immediately preceding paragraph, their respective maxima $\langle\langle V \rangle\rangle_{N,\max}$, $\langle\langle f \rangle\rangle_{N.5,\max}$, and $\langle\langle P^* \rangle\rangle_{N.5,\max}$. Averaging over $P(V)_{\text{mw}}$ — rather than over $P(V)_N$ itself — is correct to first order in V/c.

Letting S be *total* entropy, the second law is challenged by

$$\langle\langle P^* \rangle\rangle_{N.5} > 0 \implies dS/dt = -\langle\langle P^* \rangle\rangle_{N.5}/T < 0, \tag{11}$$

with maximum challenge if $\langle\langle P^* \rangle\rangle_{N.5} = \langle\langle P^* \rangle\rangle_{N.5,\max}$.

In a longer paper [12], more thorough analyses are given.

ACKNOWLEDGMENTS

Dr. Donald H. Kobe is gratefully acknowledged for very helpful and extensive discussions and correspondence. I thank Dr. Paolo Grigolini for background discussions.

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